Can Stochastic Discount Factor Models Explain the Cross Section of Equity Returns?

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Abstract

We propose a multivariate test based on no-arbitrage conditions under the stochastic discount factor approach, which compares cross-sectional variation in equity returns to the cross-sectional variation in their conditional covariance with the discount factors. Using the multivariate generalized heteroskedasticity in mean model to estimate the 25 portfolios formed on size and book-to-market ratio, together each with its own arbitrage condition, we find that the no-arbitrage test rejects the consumption-based capital asset pricing model (C-CAPM). Although the conditional covariances of returns with consumption exhibit negative variation across size, they do not vary across the book-to-market ratio. Thus, the C-CAPM can capture size effect, but not value effect. Allowing the coefficients on the consumption covariances to be different largely improves the fit of the C-CAPM, however. The value effect appears to be associated with book-to-market ratio as well as size. Book-to-market ratio separately does not generate information about average returns that cannot be explained by the C-CAPM. One possible explanation for this extra dimension of risk is the investment growth prospect of firms. Low book-to-market ratio firms may be expected to have higher rates of growth while small firms may also be expected to behave similarly.

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1. **Introduction**

Size and value effects have long been recognized as "anomalies" both in the capital asset pricing model (CAPM) literature (summarized in Fama and French, 2006 and 2008), and in the consumption-based CAPM (C-CAPM) framework with power utility (see Cochrane, 2008). This paper tests whether stochastic discount factor (SDF) models that satisfy no-arbitrage restrictions can explain the behavior of a cross section of returns on 25 portfolios sorted by firm size and their book-to-market ratio ratio (the 25 Fama-French portfolios). We examine whether portfolios of stocks have different returns due to different conditional covariances between the returns and the relevant discount factors, or because the coefficients of the discount factors vary by portfolio characteristic. This provides a test of no-arbitrage as finding either effect would imply that no-arbitrage does not hold.

Instead of modelling separate no-arbitrage conditions for the returns on the 25 Fama-French portfolios, we model them simultaneously employing an SDF framework. We use a multivariate generalized autoregressive conditional heteroskasticity in mean model (MGM) as in Smith and Wickens (2002). This methodology is in contrast to most of the time-series econometric models of equity returns in the literature, which are univariate models and do not include conditional covariances (see for example Ludvigson, 2012). Smith, Sorensen, and Wickens (2008) used the approach adopted in this paper, examining various SDF models, including the standard C-CAPM, to generate models involving macroeconomic variables. Abhakorn, Smith, and Wickens (2013) estimate the MGM for the standard C-CAPM for each of the 25 Fama-French portfolios, and find that the fit of the model is significantly improved by the inclusion of the firm book-to-market value ratio (HML) factor. This paper extends their analysis by estimating the all 25 Fama-French portfolio returns simultaneously and testing for each asset-pricing model whether the conditional covariances of these returns with the relevant discount factors can adequately explain the excess returns of these portfolios.

We find that C-CAPM is rejected by the no-arbitrage test. The model can explain the size effect, as the conditional covariance of consumption with firm size is negative, but not the value effect, as the conditional covariance of consumption with the book-to-market ratio does not vary as required across the book-to-market quintiles. We find that the value effect tends to be slightly lower for portfolios in the highest book-to-market quintile - indicating that a lower risk premium - than for portfolios with the lowest book-to-market quintiles. Allowing the coefficients on the conditional covariances to vary across the portfolios improves fit markedly. As C-CAPM restricts them to be the same, this too is an indication of the failure of the model.

The paper is set out as follows. In Section 2, we briefly review the relevant literature on CAPM and C-CAPM. In Section 3 we describe our theoretical framework for asset pricing and in Section 4 we explain our econometric methodology. In Section 5, we report our empirical results. Section 6 summarizes the findings of this paper.

2. Some relevant literature on CAPM and C-CAPM

Evidence that the accounting variables firm size and the book-to-market ratio would be significant if included in the standard CAPM in addition to the market return was first presented by Fama and French (1993 and 2008). This cast doubt on the empirical validity of the CAPM as it suggested that additional pricing factors to the market return were required to successfully explain the cross-section of stock returns. This raises the question of whether such anomalies would also be significant in alternative models to CAPM such as C-CAPM which takes into account the intertemporal nature of the investor optimization problem. Cochrane (2008) found that size and value effects are not significant in C-CAPM

More recently, however, a number of studies have attempted to explain the cross-section of equity returns using modified versions of C-CAPM that included either different or additional factors. Lettau and Ludvigson (2001) use the ratio of aggregate consumption to wealth as a conditioning variable in C-CAPM in order to better capture variations in expected returns over time. An alternative way to overcome the slowness of the consumption adjustment process was suggested by Parker and Julliard (2005) who measured the risk premium by its covariance with consumption growth cumulated over many quarters after the return period, see also Jagannathan and Wang (2007). Yogo (2006) proposed a two-factor model that includes nondurable and durable consumption growth. He found that the size and value effects are due to small and value stocks having higher durable consumption betas than large and growth stocks. Savov (2011) suggested the use of household garbage production as a proxy for consumption; as all forms of consumption produce waste, garbage growth should be informative about rates of consumption growth. These modified versions of C-CAPM seem to explain the cross-section of equity returns equally well to the Fama and French three-factor model (Fama and French, 1993). In this paper, rather than asserting that there are alternative or missing factors in C-CAPM, we exploit implications of C-CAPM that are ignored in the papers discussed above while keeping close to the ideas of Fama and French. In particular, we include the two additional factors of Fama and French, and do so using C-CAPM instead of CAPM, Thus we explore the validity of the model but in a multivariate no-arbitrage framework by estimating the 25 Fama-French portfolios simultaneously.

It appears from the results of Abhakorn, Smith, and Wickens (2013) that in order to capture the value effect using C-CAPM it is necessary to include both firm size and the book-to-market ratio as when including them individually C-CAPM cannot explain small growth portfolios. They find that HML helps explain the 25 Fama-French portfolios across size quintiles as well as across book-to-market ratio quintiles, and suggest that HML may be associated with the investment growth prospects of firm. This could be the reason why the investment-based asset pricing models of Brennan, Wang, and Xia (2004) and Li, Vassalou, and Xing (2006) are able to explain well the cross-section of equity returns but traditional CAPM is not able to (e.g. Fama and French, 1992 and 2006 and Lewellen and Nagel, 2006). This suggests that consumption contains information about these firm characteristics that is not available through market return.

3. Theoretical Framework

3.1 Stochastic Discount Factor Representations of Asset Pricing Models

The basic no-arbitrage pricing equation for a risky asset defines a relationship between the Stochastic Discount Factor (SDF) M_{t+1} and the risky return, R_{t+1}.

$$1 = E_t[M_{t+1}R_{t+1}] \tag{1}$$

where M_{t+1} is the real stochastic discount factor for period $t+1$ and for equity, the rate of return in real terms is $R_{t+1} = (P_{t+1} + D_{t+1}) / P_t$, where D_{t+1} are real dividend payments assumed to be made at the start of period $t+1$ and P_t is the real price of equity (see Cochrane, 2008). If the logarithms of M_{t+1}, R_{t+1} and the risk free rate (m_{t+1}, r_{t+1}, r_t^f) are jointly normally distributed, then (1) implies that the expected excess real return on equity is given by

$$E_t(r_{t+1} - r_t^f) + \frac{1}{2}V_t(r_{t+1}) = -Cov_t(m_{t+1}, r_{t+1}). \tag{2}$$

where the right-hand side is the risk premium and the variance term is the Jensen effect.

Equation (2) can also be expressed in terms of nominal returns. If i_{t+1} is the nominal return on equity, i_t^f is the nominal risk-free rate, P_t^c is the consumer price index, and inflation is $1 + \pi_{t+1} = P_{t+1}^c / P_t^c$, the pricing equation (1) becomes

$$1 = E_t\left[M_{t+1}(P_t^c / P_{t+1}^c)(1 + i_{t+1})\right].$$

The no-arbitrage condition for nominal returns is:

$$E_t(i_{t+1} - i_t^f) + \frac{1}{2}V_t(i_{t+1}) = -Cov_t(m_{t+1}, i_{t+1}) + Cov_t(\pi_{t+1}, i_{t+1}). \tag{3}$$

Comparing (3) and (2), the no-arbitrage condition for the nominal return involves one additional term in the conditional covariance of returns with inflation.

More generally, if m_t can be represented as a linear function of $n-1$ factors $z_{i,t}\ \{i = 1, ..., n-1\}$ so that $m_t = -\sum_{i=1}^{n-1} \alpha_i z_{i,t}$, then a general representation of (3) is

$$E_t(i_{t+1} - i_t^f) = \alpha_0 V_t(i_{t+1}) + \sum_{i=1}^{n} \alpha_i Cov_t(z_{i,t+1}, i_{t+1}), \tag{4}$$

where $z_{n,t} = \pi_t$. The differences between many asset pricing models are in their stochastic discount factor, $z_{i,t+1}$, and the restrictions imposed on the coefficients. We consider three pricing models that are special cases of equation (4):

(a) C-CAPM with power utility

The discount factor in this case is $M_{t+1} = \beta\left(C_{t+1} / C_t\right)^{-\gamma}$ where the consumer./investor has utility function, $U(C_t) = (C_t^{1-\gamma} - 1)/(1-\gamma)$ over real expenditure C_t with γ = constant coefficient of relative risk aversion (CRRA). The nominal no-arbitrage condition in this case is:

$$E_t(i_{t+1} - i_t^f) + \frac{1}{2}V_t(i_{t+1}) = \gamma Cov_t(\Delta \ln C_{t+1}, i_{t+1}) + Cov_t(\pi_{t+1}, i_{t+1}), \qquad (5)$$

where $\Delta \ln C_{t+1} \simeq \Delta C_{t+1} / C_t$ is the growth rate of consumption. C-CAPM with power utility implies that excess returns of different portfolios of equities differ due to their conditional covariance with consumption with a common CRRA.

(b) CAPM

The CAPM implies that the expected return of an asset must be linearly related to the covariance of its return with the return on the market portfolio through

$$E_t(r_{t+1} - r_t^f) = \delta_t Cov_t(r_{t+1}^m, r_{t+1})$$

where $\delta_t = E_t(r_{t+1}^m - r_t^f) / V_t(r_{t+1}^m)$ is the market price of risk and can be interpreted as the CRRA (Merton, 1980). There is no Jensen effect because log-normality is not assumed. The corresponding no-arbitrage condition for nominal returns is

$$E_t(i_{t+1} - i_t^f) = \delta_t Cov_t(i_{t+1}^m, i_{t+1}) \qquad (6)$$

where i_{t+1}^m is the nominal return on the market portfolio.

(c) General Stochastic Discount Factor Models

General SDF models are based on macroeconomic factors and particular versions of the multifactor model in (4) which also allow the factors to have unrestricted coefficients. We consider general SDF model with up to three macroeconomic factors. Smith, Sorensen, and Wickens (2008) suggest the use of factors that are associated with the business cycle and inflation. They argue that as financial institutions, such as pension funds, are the main holders of equity and act on behalf of investors and often focus on short-term business cycle considerations rather than on longer term performance associated with the utility of their investors. The authors, therefore, use output growth as an additional source of risk to consumption and inflation, but without seeking to give the model a general equilibrium interpretation.

3.2 Testing the Discount Factor Models

In all of the SDF models previously discussed, the discount factors are functions of aggregate variables, and thus it is possible to hold the properties of the discount factors constant as one individual asset is compared to another. As the risk premium is represented by the conditional covariance of the returns with the discount factor, we can compare cross-sectional average returns with cross-sectional variation in their conditional covariances with the factors. The implication is that the coefficients on these conditional covariances should be the same across the cross-section of equity returns and stocks have different returns because they have different conditional covariances with the relevant factors. Our estimation method allows these covariances also to vary through time. This relation provides testable restrictions on no-arbitrage conditions, and therefore, it can be interpreted as a no-arbitrage test.

Table 1 provides a summary of restrictions for each asset pricing models implied by its no-arbitrage condition. C-CAPM with power utility and nominal returns (M1) implies that the CRRA is constant and should be the same across the cross-section of expected returns for no arbitrage opportunities in the market. M1 is the restricted version of C-CAPM. On the other hand, allowing the coefficients on the conditional covariances of returns with consumption to be different generates an unrestricted version of C-CAPM (M4). The double-sorted, 25 size and book-to-market equity ratio portfolios generate two more versions of C-CAPM with power utility: restricted book-to-market model (M2) and restricted size model (M3). M2 allows portfolios with different size groups to have different coefficients on the consumption covariances, while M3 allows the coefficients for portfolios with different book-to-market-equity ratio groups to be different. Similarly, these restrictions of C-CAPM are applied to the CAPM, where the market price of risk is expected to be the same across assets, (M5-M8). In addition, the restricted and unrestricted general SDF models, based on two (consumption growth and inflation) and three macroeconomic variables (consumption growth, inflation and industrial production growth), are given by M9-M12. In sum, all the above asset pricing models can be represented as restricted versions of the SDF model,

$$E_t(i_{t+1}^{sb} - i_t^f) = \alpha_{0,sb} V_t(i_{t+1}^{sb}) + \alpha_{1,sb} Cov_t(\Delta c_{t+1}, i_{t+1}^{sb}) + \alpha_{2,sb} Cov_t(\pi_{t+1}, i_{t+1}^{sb})$$
$$+\alpha_{3,sb} Cov_t(\Delta q_{t+1}, i_{t+1}^{sb}) + \alpha_{4,sb} Cov_t(i_{t+1}^{m}, i_{t+1}^{sb}) \quad (7)$$

where s and b indicate size and book-to-market ratio groups that the characteristics portfolios belong to, respectively, and q_t is industrial production. The different asset models can be obtained by placing different restrictions on the α_i, *s*, and *b*.

Table 1
Restrictions on the No-arbitrage Condition

s and b indicate size and book-to-market groups for the characteristics portfolios. The numbers are in ascending order of magnitude. The smallest size is denoted by s = 1 while the lowest book-to-market ratio is represented by b = 1. γ denotes constant coefficient of relative risk aversion (CRRA). α_i represents a coefficient for each conditional covariance in Equation 8.

Models	α_0	α_1	α_2	α_3	α_4
M1: C-CAPM with power utility and nominal return	$-\frac{1}{2}$	γ	1	0	0
M2: Restricted book-to-market C-CAPM	$-\frac{1}{2}$	$\alpha_{1,s}$	1	0	0
M3: Restricted size C-CAPM	$-\frac{1}{2}$	$\alpha_{1,b}$	1	0	0
M4: Unrestricted C-CAPM	$-\frac{1}{2}$	$\alpha_{1,sb}$	1	0	0
M5: CAPM	0	0	0	0	δ
M6: Restricted book-to-market CAPM	0	0	0	0	$\alpha_{4,s}$
M7: Restricted size CAPM	0	0	0	0	$\alpha_{4,b}$
M8: Unrestricted CAPM	0	0	0	0	$\alpha_{4,sb}$
M9: Restricted two-factor SDF model	$-\frac{1}{2}$	α_1	α_2	0	0
M10: Unrestricted two-factor SDF model	$-\frac{1}{2}$	$\alpha_{1,sb}$	$\alpha_{2,sb}$	0	0
M11: Restricted three-factor SDF model	$-\frac{1}{2}$	α_1	α_2	α_3	0
M12: Unrestricted three-factor SDF model	$-\frac{1}{2}$	$\alpha_{1,sb}$	$\alpha_{2,sb}$	$\alpha_{3,sb}$	0

4. Econometric Framework

We follow the same econometric approach here as in Smith and Wickens (2002), and Smith, Sorensen, and Wickens (2008, 2010), and Abhakorn, Smith, and Wickens (2013) by using the multivariate generalized autoregressive conditional heteroskedasticity in mean model (MGM) to estimate the joint distribution of the excess return on equity with macroeconomic factors in such a way that the return satisfies the no-arbitrage condition under the SDF framework. This approach is achieved by including conditional covariances of the excess equity returns and the macroeconomic factors in the mean of the asset pricing equations and constraining the coefficients on these time-varying, conditional covariances according to the no-arbitrage condition implied by each asset-pricing model.

Let $\mathbf{x_{t+1}} = (r_{1,t+1} - r_t^f, ..., r_{i,t+1} - r_t^f, \Delta c_{t+1}, \pi_{t+1}, \Delta q_{t+1})'$ which contains n variables and i returns, as several portfolios are estimated at the same time. This specification is an extension of the MGM in Smith and Wickens (2002). Consumption, inflation, and industrial productions are included, as they give rise to the discount factors in the SDF model, M1-M12 in Table 1, through their conditional covariances with the excess returns. Additional macroeconomic variables can be included in this vector if they improve the estimate of the joint distribution. The MGM model can then be written as

$$\mathbf{x_{t+1}} = \boldsymbol{\alpha} + \boldsymbol{\Gamma}\mathbf{x_t} + \boldsymbol{\Phi}\mathbf{g_{t+1}} + \boldsymbol{\varepsilon_{t+1}},$$

where

$$\boldsymbol{\varepsilon_{t+1}} \mid I_t \sim N(0, \mathbf{H_{t+1}}),$$

$$\mathbf{g_{t+1}} = vech(\mathbf{H_{t+1}}).$$

where, $\boldsymbol{\alpha}$ is a $n \times 1$ vector of constant, $\boldsymbol{\Gamma}$ is a $n \times n$ matrix of coefficients in the vector autoregressive (VAR) part (included to obtain better representation of the error terms), $\boldsymbol{\Phi}$ is a $n \times n$ matrix of coefficients of in-mean component, $\boldsymbol{\varepsilon_{t+1}}$ is an $n \times 1$ vector of errors, and $i =$ number of equity returns. The *vech* operator converts the lower triangle of a symmetric matrix into a vector. The error term, $\boldsymbol{\varepsilon_{t+1}}$, is conditionally normally distributed with mean zero and with conditional covariance matrix $\mathbf{H_{t+1}}$. The first i rows of the model is restricted to satisfy the no-arbitrage condition as follows: 1) the first i rows of $\boldsymbol{\Gamma}$ must be zero; 2) the first i rows of $\boldsymbol{\Phi}$ depends on then specification of each asset pricing model defined in Table 1; 3) the *i+1* to *i+3* rows of $\mathbf{\Phi}$ are all zero; and 4) the first *i* elements of $\boldsymbol{\alpha}$ is zero. A likelihood ratio test is used to provide test statistics for the restrictions implied by the no-arbitrage condition in M1-M12 as given in Table 1.

While the MGM model is convenient, it is heavily parameterized, which can create numerical problems in finding the maximum of the likelihood function due to the likelihood of being relatively flat, and hence uninformative. Therefore, to complete the model parameterization for the conditional covariance matrix $\mathbf{H_{t+1}}$ with the view toward restricting the number of coefficients being estimated, the specification of the conditional covariance matrix is chosen to be the vector diagonal model with variance targeting (Ding and Engle, 2001), which can be written as follows,

$$\mathbf{H_{t+1}} = \mathbf{H_0}(\mathbf{ii}' - \mathbf{aa}' - \mathbf{bb}') + \mathbf{aa}' \odot (\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') + \mathbf{bb}' \odot \mathbf{H_t}$$

where $\odot$ denotes Hadamard product, $\mathbf{H_0}$ is the observed sample covariance matrix, and $\mathbf{a}$ and $\mathbf{b}$ are $n \times 1$ vectors. The number of parameters to be estimated reduces to $2n$. This model is particularly attractive when we estimate several excess returns simultaneously, each with its own arbitrage condition. In addition, the zero restrictions on the coefficients for excess returns in the VAR part of the macroeconomic variables are imposed to further reduce the number of parameters in the MGM model. Estimating the restricted and unrestricted C-CAPM (M1 and M4) for the 25 portfolios sorted by size and book-to-market ratio involve 69 and 93 parameters, respectively, while for the CAPM (M5 and M8) involving 53 and 78 parameters; we need to include only the market return, instead of the macroeconomic factors, in the joint distribution. We are unable to estimate M10 and M12 for the 25 portfolios, as doing so involves estimating too many parameters for our sample size (118 and 143 parameters, respectively); hence we include the two data sets of the 10 portfolios formed for size and book-to-market ratio separately to test for these general two- and three-factor SDF models, in addition to using these one-sorted portfolios to contrast the estimation results with the two-characteristics-sorted portfolios.

5. Estimation Results

5.1 Data

The data are monthly from 1960.2 to 2004.11 for the U.S. (538 observations). The return on the market portfolio is the value-weighted return on all stocks. The return on a risk-free asset is the one-month Treasury bill rate. Table 2 shows the summary statistics for the 25 value-weighted portfolios, which are the intersections of 5 portfolios formed on size and 5 portfolios formed on the ratio of book-to-market ratio. There are also two sets of ten portfolios sorted by size and book-to-market ratio separately (Table 3). All of the return variables are obtained from Kenneth French's website[b]. Real non-durable growth consumption is from the Federal Reserve Bank of St. Louis. CPI inflation and the volume index of industrial production are both from Datastream.

[b] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Table 2
Summary Statistics: 25 Size and Book-to-Market Portfolios

The table presents descriptive statistics for the excess returns on the 25 portfolios formed as the intersections of the five size and book-to-market ratio groups. Data and full definition of the returns can be found on Kenneth French's webpage. The returns are monthly value-weighted from 1960.2 to 2004.11, 538 observations. t-stat is the test statistics for zero mean hypothesis. ρ (x_t , x_{t-i}) represents the autocorrelation coefficients over the time interval i month (s).

Size Quintiles	Book-to-Market Equity Quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	Mean					Standard deviation				
Small	-0.07	0.54	0.66	0.90	0.97	8.20	6.98	5.97	5.56	5.85
2	0.10	0.47	0.72	0.82	0.89	7.48	6.07	5.36	5.14	5.73
3	0.18	0.58	0.57	0.73	0.83	6.86	5.44	4.92	4.75	5.36
4	0.34	0.38	0.63	0.75	0.70	6.04	5.15	4.83	4.61	5.35
Big	0.30	0.39	0.46	0.47	0.49	4.80	4.54	4.29	4.19	4.78
	Skewness					Excess Kurtosis				
Small	-0.53	-0.46	-0.60	-0.59	-0.58	2.72	3.38	3.72	4.35	4.20
2	-0.70	-0.89	-0.92	-0.81	-0.76	2.34	4.03	4.56	4.23	4.32
3	-0.65	-0.99	-0.95	-0.59	-0.80	2.07	4.52	3.85	3.12	4.63
4	-0.49	-0.96	-0.75	-0.32	-0.52	1.99	4.93	3.86	1.82	2.72
Big	-0.46	-0.62	-0.53	-0.15	-0.36	1.89	2.60	3.18	1.23	1.17
	Normality					t-statistics for zero mean				
Small	72.7	110.0	111.1	144.2	137.7	-0.18	1.78	2.56	3.74	3.84
2	50.7	90.1	104.7	107.4	117.5	0.29	1.81	3.13	3.72	3.60
3	44.8	94.7	79.9	84.4	124.4	0.60	2.48	2.68	3.58	3.59
4	46.9	112.3	98.4	46.5	73.3	1.29	1.73	3.02	3.77	3.05
Big	44.2	62.0	92.6	27.5	23.8	1.45	2.00	2.50	2.62	2.37
	Average firm size					Average book-to-market ratio				
Small	37	39	38	34	26	0.28	0.57	0.78	1.03	1.85
2	173	175	177	176	172	0.28	0.54	0.76	1.005	1.70
3	413	421	421	424	431	0.27	0.54	0.75	1.004	1.66
4	1068	1063	1070	1079	1075	0.27	0.55	0.75	1.03	1.70
Big	9511	7119	6166	5052	4643	0.26	0.53	0.75	1.004	1.50
	Average percent of market value					Average number of firms				
Small	0.65	0.44	0.43	0.46	0.56	492	312	315	376	603
2	0.94	0.69	0.69	0.63	0.48	152	110	109	99	77
3	1.71	1.27	1.18	1.00	0.71	115	84	78	66	46
4	3.72	2.79	2.38	1.98	1.31	97	73	62	51	34
Big	36.21	16.87	11.29	7.43	4.17	106	66	51	41	25
	$\rho(x_t, x_{t-1})$					$\rho(x_t, x_{t-3})$				
Small	0.20	0.18	0.20	0.20	0.24	-0.06	-0.09	-0.05	-0.04	-0.04
2	0.16	0.16	0.17	0.16	0.15	-0.07	-0.05	-0.05	-0.05	-0.05
3	0.12	0.15	0.16	0.16	0.14	-0.05	-0.01	-0.05	-0.02	-0.04
4	0.11	0.13	0.11	0.08	0.07	-0.04	-0.04	-0.02	0.01	-0.04
Big	0.06	0.04	0.00	-0.02	0.06	0.03	-0.01	-0.02	0.02	-0.01
	$\rho(x_t, x_{t-6})$					$\rho(x_t, x_{t-12})$				
Small	0.02	0.03	0.03	0.02	-0.01	0.00	0.02	0.06	0.08	0.13
2	0.02	0.01	0.02	0.02	-0.01	-0.03	0.03	0.05	0.08	0.10
3	0.02	0.01	0.02	-0.01	-0.01	-0.03	0.03	0.02	0.04	0.08
4	0.02	0.01	-0.03	-0.03	-0.03	-0.03	0.00	0.03	0.06	0.06
Big	-0.03	-0.06	-0.04	-0.06	0.02	0.05	0.01	0.02	0.02	0.02

Table 3
Summary Statistics: 10 Industry Portfolios and Explanatory Variables

The table presents descriptive statistics for the returns on the 10 portfolios and explanatory variables. The returns are monthly value-weighted from 1960.2 to 2004.11, 538 observations. Data and full definition of the 10 portfolios can be found on Kenneth French's webpage. $i_{m,t+1}$ and i_t^f are the returns on the market portfolios and one-month Treasury bill rate respectively. Consumption growth, inflation, and industrial production growth are represented by Δc_{t+1}, $\Delta\pi_{t+1}$, and Δq_{t+1} respectively. Std. Dev is the standard deviation. t-stat is the t-statistic for zero mean hypothesis. t-stat is the test statistics for zero mean hypothesis. ρ (x_t , x_{t-i}) represents the autocorrelation coefficients over the time interval i month(s). BM denotes book-to-market equity ratio. Firm size, book-to-market equity ratio, percent of the market, and number of firms are in average terms.

	Size Deciles									
	Small	2	3	4	5	6	7	8	9	Large
Mean	1.04	0.98	1.03	0.98	1.01	0.92	0.98	0.95	0.91	0.79
Std. Dev.	6.32	6.26	5.99	5.80	5.55	5.25	5.11	4.98	4.54	4.26
Skewness	-0.53	-0.64	-0.78	-0.86	-0.85	-0.84	-0.71	-0.64	-0.56	-0.52
Excess Kurtosis	3.23	3.52	3.22	3.56	3.42	3.20	3.33	2.49	2.45	2.13
Normality	94.89	96.69	72.24	77.02	73.66	68.42	81.48	56.85	52.81	50.40
t-stat	3.81	3.64	3.98	3.91	4.23	4.08	4.45	4.41	4.64	4.29
Firm Size	21	78	139	219	338	512	803	1346	2597	12780
% of Market	1.47	1.37	1.62	2.00	2.59	3.33	4.69	7.39	13.23	62.31
No. of firms	2123	523	347	272	229	194	175	164	152	146
$\rho(x_t, x_{t-1})$	0.24	0.17	0.16	0.16	0.14	0.01	0.12	0.09	0.08	0.01
$\rho(x_t, x_{t-3})$	-0.05	-0.07	-0.07	-0.06	-0.05	-0.04	-0.03	-0.04	-0.03	0.03
$\rho(x_t, x_{t-6})$	0.01	0.02	0.01	0.02	0.02	0.01	0.00	0.00	-0.02	-0.03
$\rho(x_t, x_{t-12})$	0.08	0.04	0.03	0.03	0.01	0.01	0.01	0.01	0.01	0.05
	Book-to-market Deciles									
	Low	2	3	4	5	6	7	8	9	High
Mean	0.67	0.85	0.87	0.86	0.93	0.99	1.05	1.09	1.12	1.21
Std. Dev.	5.24	4.77	4.73	4.66	4.35	4.36	4.26	4.26	4.64	5.33
Skewness	-0.42	-0.69	-0.81	-0.66	-0.70	-0.67	-0.17	-0.28	-0.41	-0.37
Excess Kurtosis	1.67	2.76	3.88	3.12	4.24	3.54	1.80	2.11	2.11	3.26
Normality	38.09	63.18	92.12	78.47	121.46	93.76	50.24	60.74	54.66	112.13
t-stat	2.91	4.14	4.28	4.28	4.96	5.27	5.71	5.93	5.59	5.25
Firm Size	1582	1178	981	813	723	576	527	433	349	177
BM	0.20	0.37	0.49	0.60	0.71	0.82	0.94	1.10	1.35	2.03
% of Market	32.80	15.27	11.46	8.88	7.79	6.20	5.76	4.88	4.29	2.68
No. of firms	592	370	333	312	308	307	312	322	351	433
$\rho(x_t, x_{t-1})$	0.09	0.07	0.07	0.08	0.05	0.03	0.04	0.05	0.09	0.12
$\rho(x_t, x_{t-3})$	0.02	-0.01	-0.02	-0.03	-0.04	0.01	0.02	-0.01	-0.03	-0.03
$\rho(x_t, x_{t-6})$	-0.02	-0.01	-0.04	-0.04	-0.03	0.00	-0.06	-0.03	-0.02	-0.02
$\rho(x_t, x_{t-12})$	0.02	0.02	0.01	0.01	0.02	0.02	0.01	0.07	0.06	0.07

	Explanatory variables										
	$i_{m,t+1}$	i_t^f	Δc_{t+1}	$\Delta\pi_{t+1}$	Δq_{t+1}	Correlation					
Mean	0.94	0.46	0.23	0.35	0.25		$i_{m,t+1}$	i_t^f	Δc_{t+1}	$\Delta\pi_{t+1}$	Δq_{t+1}
Std. Dev.	4.41	0.23	0.73	0.30	0.75	i_t^f	-0.04	1.00			
Skewness	-0.46	1.041	-0.04	0.99	-0.62	Δc_{t+1}	0.15	-0.09	1.00		
Excess Kurtosis	1.90	1.70	1.37	1.68	2.98	$\Delta\pi_{t+1}$	-0.14	0.54	-0.20	1.00	
Normality	44.85	98.95	33.56	82.25	75.70	Δq_{t+1}	-0.03	-0.16	0.14	-0.10	1.00
$\rho(x_t, x_{t-1})$	0.06	0.95	-0.36	0.64	0.36						
$\rho(x_t, x_{t-3})$	0.00	0.90	0.14	0.53	0.27						
$\rho(x_t, x_{t-6})$	-0.02	0.84	0.01	0.52	0.09						
$\rho(x_t, x_{t-12})$	0.02	0.72	-0.07	0.44	-0.04						

The descriptive statistics for the excess returns for the 25 portfolios in Table 2 are similar to those in Fama and French (1993 and 2006) for the periods 1963-1991 and 1963-2004 respectively, thus indicating the value effect and relatively weak size effect. This relatively weak size effect is also seen in Table 3 where one-characteristic sorted portfolios are considered. In general, all excess returns and macroeconomic variables appear to have negative skewness, excess kurtosis, and non-normality, except for the risk-free rate and inflation, which display positive skewness and show persistent volatility.

5.2 Estimates

5.2.1 C-CAPM

A full set of model estimates with their restricted versions for C-CAPM with power utility and nominal returns is reported in Table 4. A likelihood ratio test is used to examine the hypothesis implied by each restricted model against the unrestricted model, M4. For M1, the conditional covariance of returns with consumption is highly significant, but the size of the coefficient, 83.25, implies an implausibly large CRRA, which is a common feature of consumption-based models (Campbell, 2002, Cochrane, 2008, Yogo (2006), Smith, Sorensen, and Wickens, 2008, 2010) except for C-CAPM with garbage growth of Savov (2011).

For M2, all five consumption coefficients are significant, and the likelihood ratio rejects the hypothesis that portfolios within different book-to-market equity ratio quintiles have the same coefficient at any conventional levels. The test statistic for the restrictions against the unrestricted model M4 is close to that for M1, implying that the differences in the coefficients across size have little weight on the behavior of the estimated returns. On the other hand, the likelihood ratio test marginally fails to reject M3 ($p-value=0.0513$); restricting the consumption coefficients for portfolios within the same size quintiles to be the same does not exclude significant information about the excess returns. In other words, size has no, or a relatively weak, relation to the consumption coefficient. In fact, the coefficients in M2 for each size quintile look very similar, while those in M3 for each book-to-market equity ratio quintiles increase substantially from the lowest to the highest book-to-market quintiles. In addition, the consumption covariance coefficient for the lowest book-to-market equity ratio quintiles in M3 is not significantly different from zero.

M4 has 23 coefficients that are significant at conventional significance levels. All coefficients range widely from 47.79 to 247.14. Looking down each column, there is no clear pattern in the values of the coefficients across the size quintiles, whilst when looking across each row; the coefficients tend to rise as the book-to-market ratio increases. Like the insignificance of the consumption coefficient on the lowest book-to-market quintile in M3, the remaining 2 coefficients for the lowest book-to-market quintile and the first 2 smallest size quintiles are not significantly different from zero. The insignificance of the coefficients for the lowest book-to-market quintiles of the 25 portfolios is consistent with the evidence from other empirical asset pricing studies on the 25 portfolios (Fama and French, 2008, Lettau and Lugvigson, 2001, Parker and Julliard, 2005, Yogo, 2006, and Savov, 2011) where the pricing models they propose also have difficulty explaining the portfolios in the smallest size and lowest book-to-market quintiles (small growth portfolio). This inability may be due to limits to arbitrage from short-sale constraints for these portfolios, and thus frictionless equilibrium models, including C-CAPM, cannot explain the returns on these small growth portfolios (Yogo, 2006).

Figure 1 shows a scatter plot of average actual and estimated excess returns in M1 to M4 for the 25 portfolios. If the pricing model fits the data well, the points should all lie on a 45-degree line. In Figure 1(d), M4 appears to best explain the excess return on these portfolios and is more or less as good as the modified versions of C-CAPM and the Fama and French three-factor model. The differences in the estimated risk premium and actual excess return ranges from 0.01% to 0.18% per month, which is lower than those in M1-M4.

Figures 1(a) and 1(b) show that the estimated risk premia from M1 and M2 are similar, implying that imposing the restrictions on size quintiles does not affect the behavior of risk premia. On the other hand, allowing the consumption coefficients to be different, as in M3, improves the performance of the model sharply, except for the 5 portfolios in the lowest book-to-market ratio quintiles. This observation suggests that book-to-market equity ratio seems to have additional information about the average excess returns that is not captured by C-CAPM.

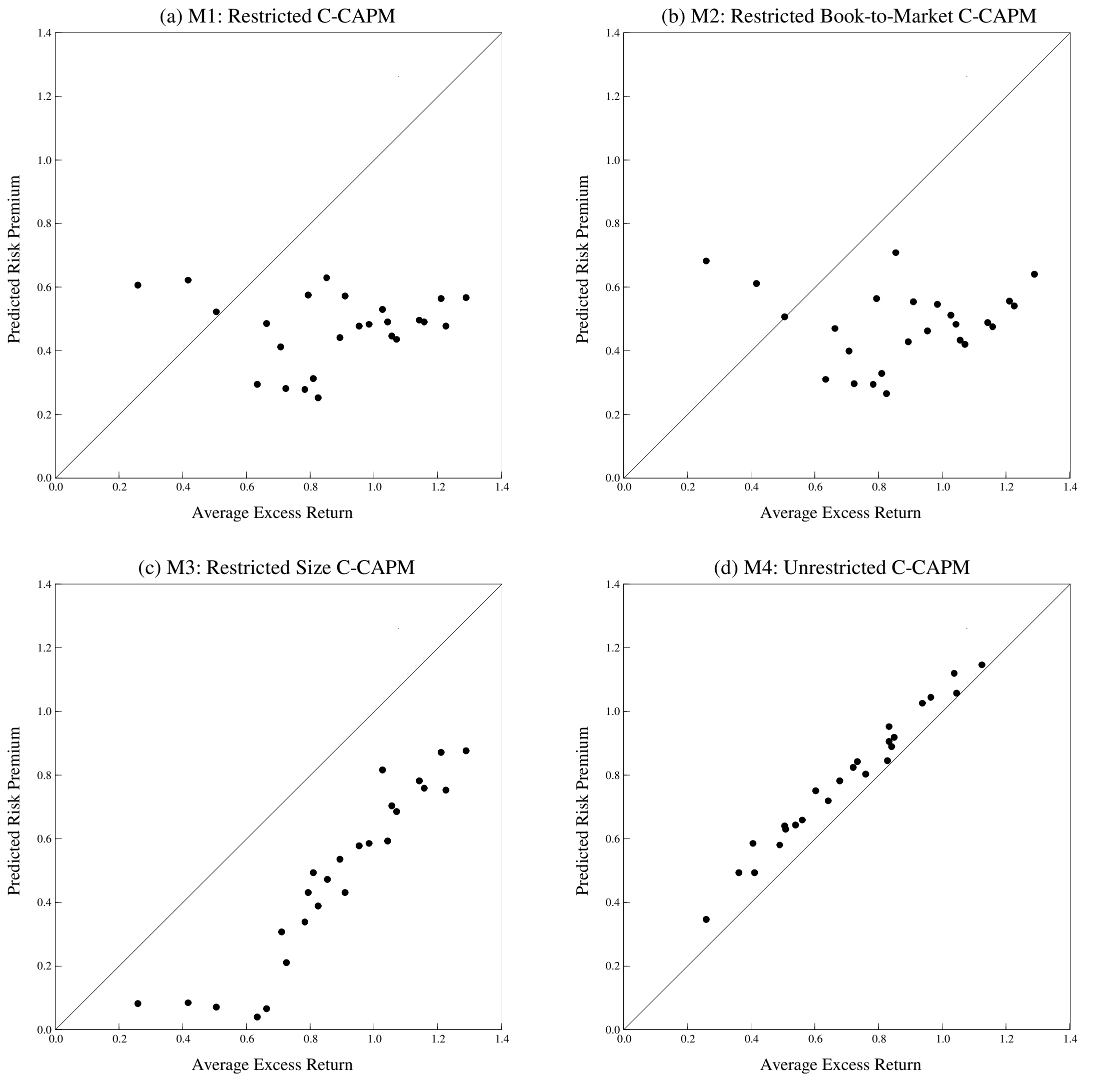
Table 4
Estimates of C-CAPM

The table presents the estimates of the C-CAPM (M1-M4): 1960.2-2004.11, 538 observations. γ denotes the coefficient relative risk aversion and α_i represents a coefficient for each conditional covariance in Equation 8. $t(\gamma)$ and $t(\alpha_i)$ are their corresponding t–statistics respectively. The pricing models (M1-M4) are tested against each other using the log-likelihood ratio test. 2Δlog represents the likelihood ratio statistic. The corresponding p-value at 5% significance level is denoted by *p-value*.

Panel A: 25 Size and Book-to-Market Portfolios										
	Panel A1: Restricted C-CAPM (M1)									
	γ	$t(\gamma)$	$2\log$	$p-value$						
	83.25	4.11	89.30	0.0000						
	Panel A2: Restricted Book-to-Market C-CAPM (M2)									
	Size Quintiles									
	Small	2	3	4	Big					
α_{1s}	93.83	81.83	80.50	80.57	87.62					
$t(\alpha_{1s})$	4.13	3.89	3.73	3.58	2.62					
$2\log$	87.27									
$p-value$	0.0000									
	Panel A3: Restricted Size C-CAPM (M3)									
	Book-to-Market Quintiles									
	Low	2	3	4	High					
α_{1b}	11.49	62.46	100.51	130.99	128.32					
$t(\alpha_{1b})$	0.40	2.76	4.31	5.53	5.64					
$2\log$	31.31									
$p-value$	0.0513									
	Panel A4: Unrestricted C-CAPM (M4)									
Size	Book-to-Market Quintiles									
Quintiles	Low	2	3	4	High	Low	2	3	4	High
			α_{sb}					$t(\alpha_{sb})$		
Small	47.79	106.31	145.27	183.46	167.67	0.90	2.55	3.32	4.47	4.66
2	66.32	104.24	155.03	171.43	164.63	1.46	2.76	4.07	4.70	4.61
3	93.06	119.64	146.79	177.21	176.68	1.86	3.55	3.73	4.65	4.45
4	108.03	129.48	146.63	169.83	142.16	2.21	2.82	3.97	4.44	3.83
Big	139.19	171.24	190.97	175.10	247.14	2.16	2.75	3.22	3.35	3.43
Panel B: 10 Size Portfolios										
	Panel B1: Restricted C-CAPM (M1)									
	γ	$t(\gamma)$	$2\log$	$p-value$						
	81.96	3.29	16.71	0.0534						
	Panel B2: Unrestricted C-CAPM (M4)									
	Size Deciles									
	Small	2	3	4	5	6	7	8	9	Large
α_{1s}	141.11	121.66	133.98	124.19	133.63	143.53	137.74	153.18	176.75	181.85
$t(\alpha_{1s})$	3.88	3.68	4.06	3.96	4.32	4.26	4.32	4.31	4.44	4.01
Panel C: 10 Book-to-Market Portfolios										
	Panel C1: Restricted C-CAPM (M1)									
	γ	$t(\gamma)$	$2\log$	$p-value$						
	261.78	7.93	5.96	0.7439						
	Panel C2: Unrestricted C-CAPM (M4)									
	Book-to-Market Deciles									
	Low	2	3	4	5	6	7	8	9	High
α_{1b}	215.24	235.78	249.82	252.10	267.28	270.21	272.45	279.32	254.01	250.89
$t(\alpha_{1b})$	4.18	5.42	5.51	5.88	5.89	6.72	7.44	7.82	6.99	7.33

Figure 1
Cross-Sectional Fit: C-CAPM for the 25 Size and Book-to-Market Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 25 size and book-to-market portfolios. The estimated models are (a) M1, (b) M2, and (c) M3, and (d) M4. The average excess returns are adjusted for the Jensen effect.



We estimate M1 and M4 for the two one-characteristic sorted portfolios to investigate whether these provide similar information to the double-sorted portfolios. Panels B and C in Table 4 show that C-CAPM with power utility performs better, as neither of the M1 is rejected, suggesting that sorting stocks according to size and book-to-market ratio may more accurately distinguish stocks. For the 10 size portfolios, all coefficients on the consumption covariances in M4 are significant, and the model fits the data better than M1 (Figure 2). This is also true for the 10 book-to-market ratio portfolios (Figure 3). In addition, the consumption coefficient for the portfolio in the lowest book-to-market quintile is highly significant, while those for the small growth portfolios in the 25 portfolios are not. The descriptive statistics in Table 2 show that the average book-to-market ratios for these two portfolios are similar, while their average firm sizes are very different. Firms in the smallest size and the lowest book-to-market quintiles seem to be much smaller than other firms in the lowest book-to-market quintiles. Therefore, additional information that is not captured by C-CAPM may be associated with both book-to-market equity ratio as well as size. One possible explanation is that this extra dimension of risk arises from the investment growth prospect of firms. Abhakorn, Smith, and Wickens (2013) find that, in the C-CAPM framework, the mimicking return factor related to book-to-market ratio (HML) can explain the 25 Fama-French portfolios across size quintiles as well as across book-to-market ratio quintiles. In this regard, they assert that HML may represent risk associated with the investment growth prospects of firm as low book-to-market ratio firms may be expected to have higher rates of growth while small firms may also be expected to behave similarly. This interpretation of the extra dimension of risk is also consistent with Brennan, Wang, and Xia (2004) and Li, Vassalou, and Xing (2006). These two studies propose asset pricing models based on investment related factors that can explain the cross-section of equity returns well.

Figure 2
Cross-Sectional Fit: C-CAPM for the 10 Size Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 10 size portfolios. The estimated models are (a) M1 and (b) M4. The average excess returns are adjusted for the Jensen effect.

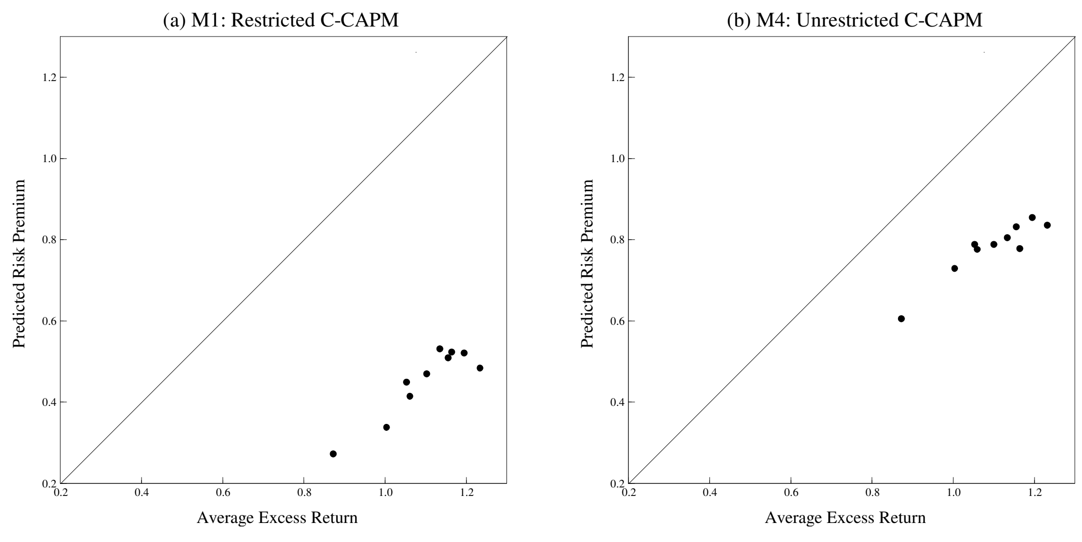
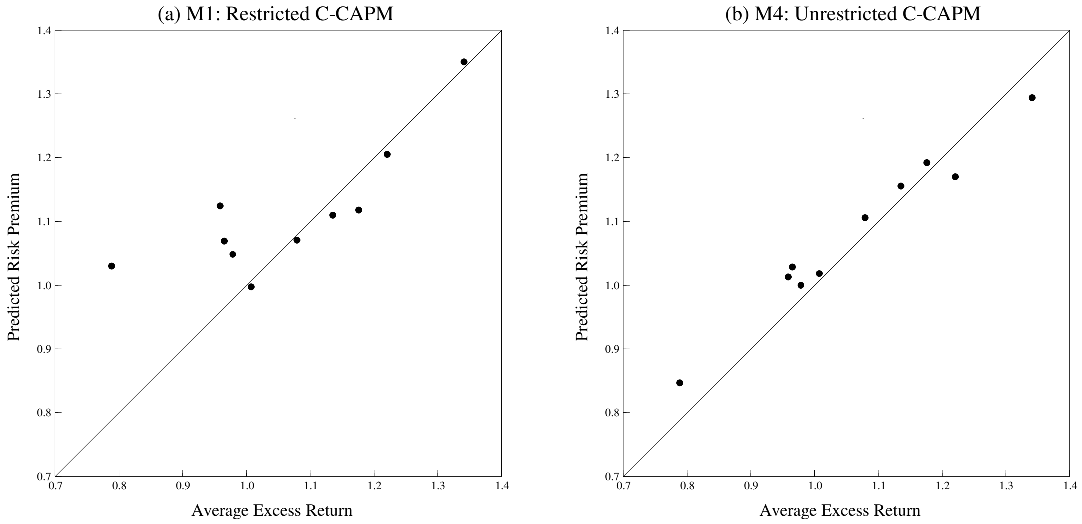


Figure 3
Cross-Sectional Fit: C-CAPM for the 10 Book-to-Market Ratio Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 10 book-to-market ratio portfolios. The estimated models are (a) M1 and (b) M4. The average excess returns are adjusted for the Jensen effect.



5.2.2 CAPM

Table 5 reports the estimation results for all versions of CAPM. The market price of risk in M5 is 2.77 with a t-statistics of 2.94 and is lower than those reported in earlier related studies for the U.S. market (Harvey (1989) and Ng (1991)). Comparing M6 to M2, where the coefficients for the first three size quintiles are relatively less significant, suggests that CAPM cannot price relatively small portfolios well and that there is more information in these portfolio returns related to size left unexplained by CAPM than by C-CAPM discussed above. This inability to price small portfolios has nothing to do with the book-to-market ratio. Moreover, as in M3, the coefficient on the market return for the portfolios in the lowest book-to-market ratio in M7 is not significant.

Table 5
Estimates of the CAPM

The table presents the estimates of the CAPM (M5-M8): 1960.2-2004.11, 538 observations. δ denotes the market price of risk and α_i represents a coefficient for each conditional covariance in Equation 8. $t(\delta)$ and $t(\alpha_i)$ are their corresponding t–statistics respectively. The pricing models (M5-M8) are tested against each other using the log-likelihood ratio test. 2Δlog represents the likelihood ratio statistic. The corresponding p-value at 5% significance level is denoted by *p-value*.

Panel A: 25 Size and Book-to-Market Portfolios

Panel A1: Restricted CAPM (M5)

δ	$t(\delta)$	$2\log$	$p-value$
2.77	2.94	153.72	0.0000

Panel A2: Restricted Book-to-Market CAPM (M6)

	Size Quintiles				
	Small	2	3	4	Big
α_{4s}	1.70	1.69	1.74	2.03	2.61
$t(\alpha_{4s})$	1.56	1.68	1.80	2.11	2.79
$2\log$	110.22				
$p-value$	0.0000				

Panel A3: Restricted Size CAPM (M7)

	Book-to-Market Quintiles				
	Low	2	3	4	High
α_{4b}	0.93	2.54	3.51	4.57	4.73
$t(\alpha_{4b})$	0.93	2.68	3.75	4.79	4.89
$2\log$	43.04				
$p-value$	0.0020				

Panel A4: Unrestricted CAPM (M8)

Size Quintiles	Book-to-Market Quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	α_{4sb}					$t(\alpha_{4sb})$				
Small	0.02	2.20	3.02	4.37	4.51	0.02	1.75	2.59	3.78	3.93
2	0.76	2.30	3.55	4.36	4.34	0.67	2.16	3.40	4.10	4.02
3	1.30	3.05	3.14	4.37	4.26	1.15	3.00	3.14	4.26	3.92
4	1.94	2.26	3.59	4.16	3.57	1.74	2.26	3.53	4.04	3.27
Big	1.93	2.22	2.70	3.20	3.21	1.81	2.15	2.53	2.91	2.63

Panel B: 10 Size Portfolios

Panel B1: Restricted CAPM (M5)

δ	$t(\delta)$	$2\log$	$p-value$
-0.47	-0.34	811.30	0.0000

Panel B2: Unrestricted CAPM (M8)

	Size deciles									
	Small	2	3	4	5	6	7	8	9	Large
α_{4s}	3.57	3.07	3.41	3.13	3.44	3.27	3.29	3.26	3.23	2.45
$t(\alpha_{4s})$	3.46	3.27	3.74	3.44	3.93	3.69	3.82	3.80	3.77	2.88

Panel C: 10 Book-to-market Portfolios

Panel C1: Restricted CAPM (M5)

δ	$t(\delta)$	$2\log$	$p-value$
1.98	1.79	404.85	0.0000

Panel C2: Unrestricted CAPM (M8)

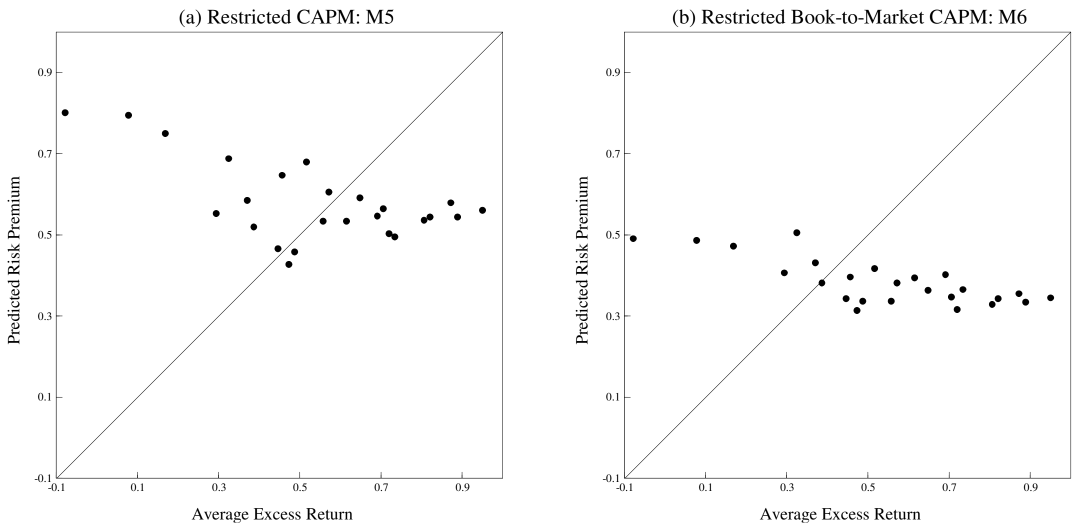
	Book-to-market Deciles									
	Low	2	3	4	5	6	7	8	9	High
α_{4b}	3.14	4.29	4.68	4.76	5.50	5.73	6.64	6.96	6.18	6.24
$t(\alpha_{4b})$	3.19	4.73	5.39	5.51	6.06	6.58	7.76	7.97	6.86	7.04

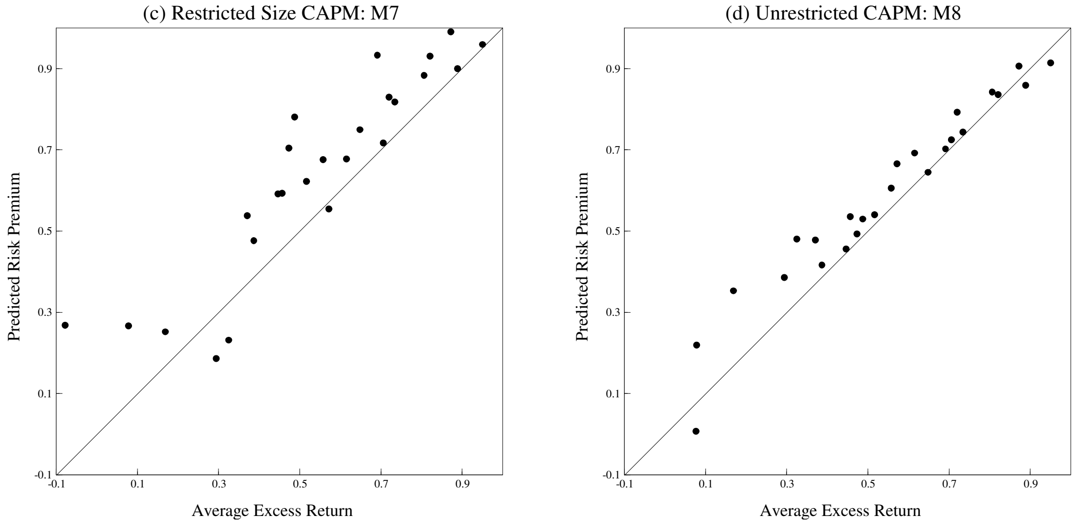
M5, M6, and M7 are rejected relative to the unrestricted model M8 based on their likelihood ratio statistics of 153.72, 110.22, and 43.04 respectively. The likelihood ratio statistics for CAPM are all larger than those for C-CAPM. As in C-CAPM, allowing the coefficients on conditional covariances of market return with individual excess return to be different offers extra information about the cross-section of the equity returns. As can be seen from Figure 4, M8 can explain the variation in the cross section well while M5 is not able to explain both size and value effects as in previous studies (e.g. Fama and French , 1992 and 2006 and Lewellen and Nagel, 2006).

In M8, 20 coefficients on conditional covariances of returns with the market return, including one for the market return (the coefficient on the variance), are more than 2 standard errors different from zero, while the other 3 coefficients are significant at the 10% significance level. The coefficients for the first 3 size and lowest book-to-market quintiles are insignificant at any conventional level. These coefficients range from 0.02 to 4.51, exhibiting a clear positive relation with book-to-market ratio. However, any relation between the coefficients and size can be seen only from the portfolios in the last two book-to-market quintiles.

Figure 4
Cross-Sectional Fit: CAPM for the 25 Size and Book-to-Market Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 25 size and book-to-market portfolios. The estimated models are (a) M5, (b) M6, and (c) M7, and (d) M8. The average excess returns are adjusted for the Jensen effect.





M5 for the two sets of 10 portfolios does not perform well since the market price of risk is of relatively low significance and in the case of the 10 size portfolios, the market price of risk has the wrong sign. Therefore, the likelihood ratio test rejects M5 for both 10 portfolios. M8 fits the data better than M5 (Figures 5 and 6), but its coefficients exhibit a negative relation with size and a negative relation with book-to-market ratio for both 10 portfolios as in the case of the 25 portfolios. On the other hand, the relation between the consumption coefficients and firm characteristics in C-CAPM can be seen only in the case of the 25 portfolios. Thus, C-CAPM can explain size effect, but it has difficulty explaining the value effects; this exposure to the value premium appears to be associated with both the book-to-market ratio and, to some extent, with size.

Figure 5
Cross-Sectional Fit: CAPM for the 10 Size Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 10 size portfolios. The estimated models are (a) M5 and (b) M8. The average excess returns are adjusted for the Jensen effect.

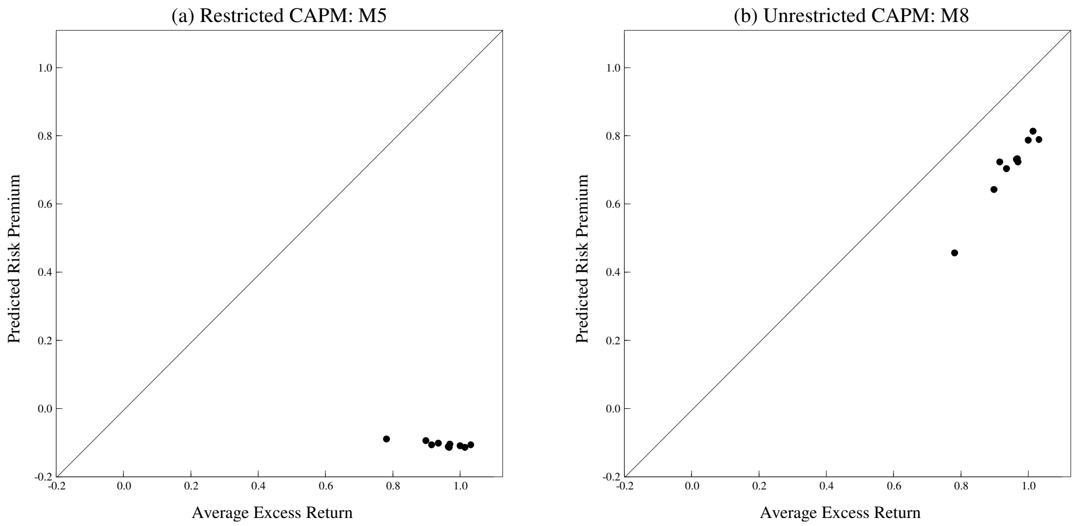
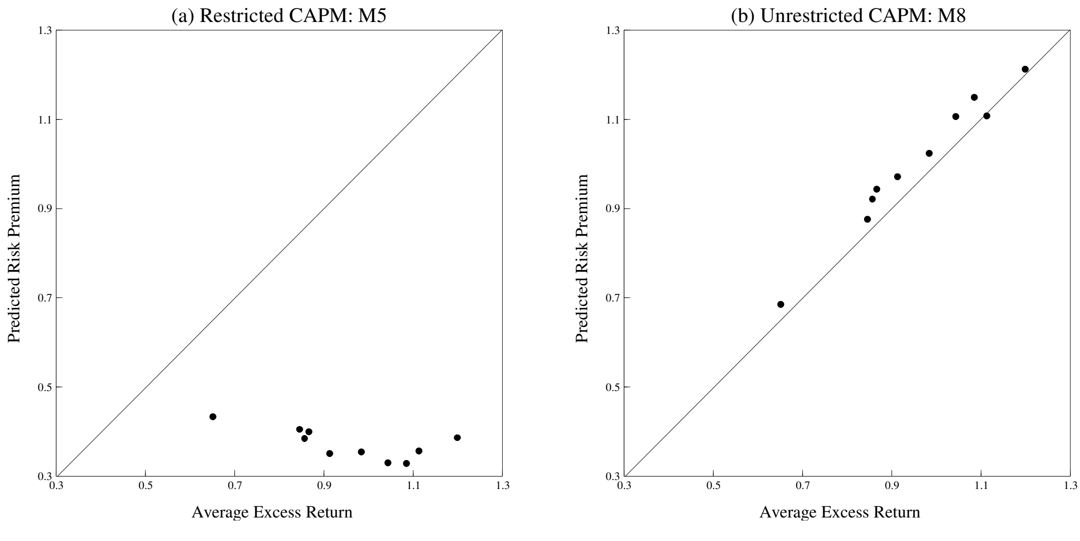


Figure 6
Cross-Sectional Fit: CAPM for the 10 Book-to-Market Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 10 book-to-market ratio portfolios. The estimated models are (a) M5 and (b) M8. The average excess returns are adjusted for the Jensen effect.



5.2.3 C-CAPM and CAPM

We further investigate the ability of C-CAPM (M1) and CAPM (M4) by comparing the behavior of the conditional covariances in both models, as given in Table 6 below, with the average excess returns of the 25 portfolios shown in Table 2. The consumption covariance obtained from the estimation of C-CAPM decreases, as we move down the column, indicating a negative relation with size. Thus, C-CAPM can capture the size effect. However, in the lowest book-to-market quintiles, average excess returns increase as the size of the portfolios grows larger. This result is consistent with the results in Table 4 where the coefficients for these portfolios in M3 and M4 are not significant from zero.

Table 6
Conditional Covariances of the Returns on the 25 Portfolios in M1 and M5

The table shows the average conditional covariances of the returns with consumption and market returns as implied by the C-CAPM and CAPM, respectively. Each conditional covariance is given in percent per month and estimated by the multivariate GARCH in the mean.

Size Quintile	Book-to-market ratio Quintile				
	Low	2	3	4	High
	Panel A: Conditional covariance of consumption				
Small	0.32218	0.28800	0.31130	0.22016	0.27699
2	0.24066	0.23827	0.23237	0.17594	0.20899
3	0.19671	0.17586	0.16247	0.15597	0.17918
4	-0.00131	-0.00166	-0.00138	0.00593	0.00496
Big	0.00304	0.00001	-0.00133	-0.00127	-0.00040
	Panel B: Conditional covariance of market return				
Small	0.32143	0.35714	0.25829	0.25272	0.28785
2	0.24282	0.22963	0.21088	0.24303	0.17777
3	0.23056	0.18894	0.18875	0.17934	0.16103
4	0.18551	0.21742	0.20223	0.15693	0.20226
Big	0.27074	0.24804	0.19669	0.16809	0.16478

C-CAPM appears to miss the value premium completely by not producing dispersion in the consumption covariance across the book-to-market quintiles. In fact, the consumption covariances for the 5 portfolios in the highest book-to-market quintile seem to be slightly lower than those in the lowest book-to-market quintiles, indicating lower risk premium is implied by C-CAPM. On the other hand, the condition covariances of the returns with the market returns in CAPM, in addition to having a similar behavior across book-to-market quintiles as consumption covariances, appear not to be able to capture the size effect as well. The dispersion of the market covariances is not big enough to explain the differences in the excess returns across size, confirming our previous results where the coefficients for the first two size quintiles in M6 were not highly significant.

We add a constant term in M1-M8 for the estimation of the 25 portfolios to measure variation in excess returns that was left unexplained in each model. In general, we expect the constant term in C-CAPM to be of more significance than in those in CAPM because pricing asset returns with market return is expected to be more precise than using aggregate consumption data. However, Table 7 shows that for the magnitude of the constant terms in CAPM, 1.98 is larger than that for C-CAPM at 0.86. This larger magnitude of CAPM is present in every restricted version. The magnitude of the constant is also larger than in Fama and French (1993) with a constant for the CAPM being 0.04 to 0.57 (in absolute terms).

Table 7
Constant term

The table presents the estimates for the constant term, in all versions of the C-CAPM and CAPM, M1-M8 in Table 1, for the 25 portfolios formed based on size and book-to-market ratio. The number in the parentless is the t-statistic associated with each constant term.

Panel A: C-CAPM				
	M1	M2	M3	M4
Constant	0.84	1.14	0.52	0.87
	(5.36)	(6.39)	(2.75)	(1.75)
Panel B: CAPM				
	M5	M6	M7	M8
Constant	1.98	1.75	0.56	0.94
	(9.13)	(6.79)	(1.85)	(1.70)

The information about the cross-section of equity returns left unexplained in C-CAPM seems to be less than that in the CAPM. Moving from M1 to M4 decreases the significance of the constant terms (except for moving from M1 to M2), suggesting that allowing coefficients of conditional covariances within the same book-to-market ratio to be different is more important than allowing the coefficients to be different across size quintiles; the magnitude and level of significance of the constant terms reduces more when moving from M2 to M3 than when moving from M1 to M2. This argument is also true for CAPM when moving from M5 to M8.

5.2.4 General SDF Models

Table 8 reports the estimates of the general two- and three-factor SDF models based on consumption, inflation, and industrial production. We are unable to estimate M10 and M12 for the 25 portfolios due to the high parameterization of the MGM. As in Smith, Sorensen, and Wickens (2008), we find that industrial production plays no role in evaluating asset returns, but inflation is significant. The coefficient on the conditional covariance of inflation for the 10 book-to-market portfolios is positive because the contribution to the risk premium by consumption is higher than it is for actual excess return. The estimation of M9 and M11 for both 10 portfolios shows that the restrictions they provide on M10 and M12 cannot be rejected by the likelihood ratio test, implying that the coefficients for conditional covariance of consumption and inflation with the returns are similar across size and book-to-market deciles. However, M9 does not explain the data better than C-CAPM and the CAPM (Figure 7).

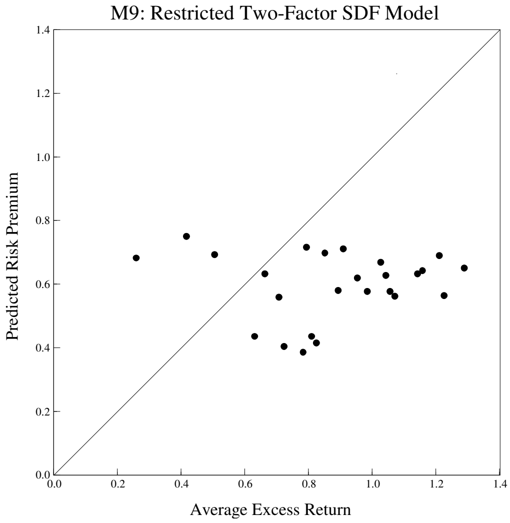
Table 8
Estimates for Restricted Macro SDF Models

The table presents the estimates for the restricted general SDF models (M9 and M11): 1960.2-2004.11, 538 observations. α_i represents a coefficient for each conditional covariance in Equation 8. $t(\alpha_i)$ is its corresponding t–statistics respectively. The pricing models (M9 and M11) are tested against their respective unrestricted alternatives (M10 and M12) using the log-likelihood ratio test. 2Δlog represents the likelihood ratio statistic. The corresponding p-value at 5% significance level is denoted by *p-value*.

Model	α_1	α_2	α_3	$2\log$	$p-value$
Panel A: 25 Size and Book-to-Market Portfolios					
M9	72.93 (3.40)	-115.72 (-1.74)			
M11	72.82 (3.40)	-116.47 (-1.73)	1.42 (0.06)		
Panel B: 10 size portfolios					
M9	59.29 (2.18)	-130.87 (-2.01)		23.66	0.1666
M11	59.62 (2.17)	-137.76 (-2.08)	16.84 (0.55)	33.43	0.1832
Panel C: 10 Book-to-market portfolios					
M9	307.12 (8.00)	147.29 (1.92)		23.27	0.1805
M11	305.17 (7.87)	141.17 (1.82)	8.58 (0.35)	32.33	0.2201

Figure 7
Cross-Sectional Fit: Two-Factor SDF Model for the 25 Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 25 size and book-to-market ratio portfolios. The estimated model is M9. The average excess returns are adjusted for the Jensen effect.



6. Conclusion

This paper examines the behavior of the cross-section of equity returns based on the no-arbitrage condition derived from the Stochastic Discount Factor approach to asset pricing. We test whether the conditional covariances of the equity returns across portfolios formed on size and book-to-market ratio with discount factors in each asset pricing model can sufficiently explain the excess returns in these portfolios. Our results indicate that the no-arbitrage test rejects C-CAPM as the model can explain the size effects, but not the value effect. Although the consumption covariances exhibit a negative relation with size, but they do not vary with the book-to-market ratio. This behavior explains why the likelihood ratio test indicates that the coefficients for the consumption covariances are not similar across book-to-market ratios.

Allowing the coefficients on the conditional covariances with consumption to be different across portfolios generally improves the fit of C-CAPM. Even without adding any factor to the model, the performance of the resulting unrestricted C-CAPM is comparable to the modified version of C-CAPM of Lettau and Ludvigson (2001), Parker and Julliard (2005), Yogo (2006) and Savov (2011). The unexplained variation in excess returns is less than for unrestricted C-CAPM as the significance of the constant term is lower than that in standard C-CAPM. Unrestricted C-CAPM does not explain the small growth portfolios well, but this phenomenon is common to most asset pricing models.

Our results confirm the findings of Abhakorn, Smith, and Wickens (2013) that both firm size and the book-to-market ratio need to be included in the model in order discover a value effect. Firm size or the book-to-market ratio on their own does not generate information about average returns that improves on C-CAPM. This requires the double sorting of stocks according to size and the book-to-market ratio for the 25 Fama-French portfolios. This finding suggests that there is an additional dimension of risk left unexplained by C-CAPM or by an SDF model with only one of these factors . This extra dimension of risk seems to be associated with both (small) size and a (low) book-to-market ratio. A possible explanation for this extra dimension of risk is the investment growth prospect of firms, see Abhakorn, Smith, and Wickens (2013), and could be the reason that the investment-based asset pricing models of Brennan, Wang, and Xia (2004) and Li, Vassalou, and Xing (2006) are able to explain the cross-section of equity returns.

Our results indicate that C-CAPM with size and the book-to-market ratio as additional factors contains information about cross-section average returns that is not captured by CAPM in previous studies by, for example, Fama and French (1992 and 2006) and Lewellen and Nagel (2006). In general, SDF models suggest that inflation seems to be significant in determining stock returns, but industrial production plays no role in determining stock returns. However, pricing models that include inflation do not perform better than C-CAPM.

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